

Load flow problems cont. ...

$$\frac{\bar{S}_k}{\bar{V}_k} = \sum_{j=1}^n \bar{Y}_{kj}^* \bar{V}_j^* \quad (8)$$

where $\bar{S}_k = P_k + jQ_k = \bar{V}_k \bar{I}_k^*$

$$\bar{S}_k = \bar{V}_k \sum \bar{Y}_{kj}^* \bar{V}_j^* \quad 8 \text{ revised.}$$

equation 8 is non linear (quadratic in \bar{V}) and the solution, in general, requires an iterative approach. This is the basic load flow equation. Equation 8 is complex and can be rewritten in Polar form as:

$$(9) \quad (A) \quad P_k = \sum_{j=1}^n |\bar{V}_k \bar{Y}_{kj} \bar{V}_j| \cos(\delta_k - \delta_j - \theta_{kj})$$

$$(B) \quad Q_k = \sum_{j=1}^n |\bar{V}_k \bar{Y}_{kj} \bar{V}_j| \sin(\delta_k - \delta_j - \theta_{kj})$$

or in rectangular form

$$(10) \quad (A) \quad P_k = \operatorname{Re} [e_k + jf_k] \sum_{j=1}^n (G_{kj} - B_{kj})(e_j - jf_j)$$

$$(B) \quad Q_k = \operatorname{Im} [e_k + jf_k] \sum_{j=1}^n (G_{kj} - B_{kj})(e_j - jf_j)$$

where

$$\bar{V}_k = |\bar{V}_k| \angle \delta_k = (e_k + jf_k)$$

$$\bar{Y}_{kj} = |\bar{Y}_{kj}| \angle \theta_{kj} = (G_{kj} - jB_{kj})$$

Equation 9

In equation 9, there are 4 quantities at each bus, assuming.

$P, Q, |V|$, and δ .

Normally, two are specified and the objective is to evaluate system voltages and phase angles to satisfy these equations.

Buses are classified according to specifications

1) Load Bus

P, Q specified

$|V|, \delta$ unknown

2) Voltage controlled bus

$P, |V|$ specified

Q, δ unknown

3) Slack Bus (AKA swing bus)

$|V|, \delta$ specified

P, Q unknown.

note a slack bus is chosen, usually as a reference bus with $\delta = 0^\circ$.

Solution Methods:

There are many different iterative techniques available to power system engineers. The nodal admittance matrix methods are, in general, superior to impedance or loop impedance methods. Two of the most commonly-used approaches for solutions are the Gauss-Seidel and the Newton-Raphson techniques.

Gauss-Seidel (GS) is easy to program and as long as the diagonal terms of the Y matrix are dominant, the approach behaves well. Its convergence is linear and slow.

The Newton-Raphson (NR) method is more complex but has a faster (quadratic) convergence, so that fewer iterations are required in spite of having more computations per iteration. It is faster and the preferred approach. For example a 500 bus problem may take 500 GS, but only 4 NR iterations, with time advantage of 15:1. However, NR takes more storage and is sensitive to starting conditions.

GAUSS-SEIDEL

Consider the two non-linear systems given by.

$$2x_1 + x_1x_2 - 1 = 0 \quad (1)$$

$$2x_2 + x_1x_2 + 1 = 0 \quad (2)$$

Before we start we know $x_1 = 1$, $x_2 = 2$

Rearranging eqn 1 & 2 we get,

$$x_1 = 0.5 - \frac{x_1x_2}{2}$$

$$x_2 = -0.5 + \frac{x_1x_2}{2}$$

Make an initial guess of $x_1(0) = 0$
 $x_2(0) = 0$

Iteration #1

$$^{(1)}x_1 = 0.5 - \frac{0}{2} = 0.5$$

$$^{(1)}x_2 = -0.5 - \frac{0}{2} = -0.5$$

Iteration #2

$$^{(2)}x_1 = 0.5 - \frac{(0.5)(-0.5)}{2} = 0.625$$

$$^{(2)}x_2 = -0.5 + \frac{(0.625)(-0.5)}{2} = -0.656$$

Iteration #3

$$^{(3)}x_1 = 0.5 - \frac{(0.625)(-0.625)}{2} = 0.705$$

$$^{(3)}x_2 = -0.5 + \frac{(0.705)(-0.656)}{2} = -0.731$$

etc $x_1 \rightarrow 1$ $x_2 \rightarrow -1$

note: GS updates with most recent values.